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A MODAL APPROPRIATION BASED METHOD FOR OPERATIONAL MODAL ANALYSIS

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ABSTRACT

In this paper, a modal appropriation methodology is suggested for Operational Modal Analysis. The method is based on performing a numerical convolution of a sine wave force with correlation functions of observed output responses of the structure. Classical modal appropriation based techniques are then used to estimate modal parameters characterizing the dynamics of the structure. The paper explains the theory and steps associated with this method and its performance is illustrated by means of studies conducted on an analytical 15 DOF system. Further, the results are compared with those obtained using standard OMA algorithms such as Stochastic Subspace Identification (SSI) algorithm.

Keywords: Modal Appropriation, Stochastic subspace identification, modal parameter estimation

1. INTRODUCTION

Classical modal parameter identification methods are usually based on frequency response functions or impulse response functions that require measurements of the input force and the resulting response. However, in some practical situations, modal parameters must be extracted from response measurements only. For example, for large structures (such as bridges, offshore platforms, and wind turbines), it is very difficult and sometimes impossible to measure actual excitation (such as wind, road noise, and wave excitation). The large amount of energy necessary to induce structural vibrations may cause local damage and excitation becomes very difficult to generate. Moreover, the actual operating conditions may differ significantly from the laboratory conditions. Therefore, in these applications, the system identification approach must be done based on in-operation output-only data. The method of in-operation modal analysis has gained considerable attention in recent years. There have been several different approaches to estimate modal parameters from output-only data. They include peak-picking from power spectral density functions, Least Squares curve fitting technique, subspace methods and the natural excitation technique (NExT) using cross correlation functions instead of impulse response functions.

On the other hand, one of the most powerful methods for modal identification is the modal (or force) appropriation method which uses one sine signal to generate forces at different points of the structure and adjusts the relative values of those forces so as to isolate a single mode. Such tests provide very accurate information on the modeshapes, which is then complemented by specific tests to determine the modal damping.

Inspired by this last method, we propose in this paper a modal appropriation based method for use with in-operation modal analysis (INOPMA). The key idea is the realization that the correlation sequence of the outputs of a vibrating structure may be considered as an impulse response but with a certain phase shift [1]. By taking the convolution of one sine wave with the correlation sequence, we show that a mode is isolated at a characteristic frequency that depends on the damping ratio. By using a force that is in quadrature of phase with the first one, the damping ratio may be estimated which in turn leads to the estimation of the undamped natural frequency. The algorithm is described in section 3 and its performance on an analytical system is compared with Stochastic Subspace Identification (SSI) algorithm in section 4. The SSI algorithm is also described briefly in section 2.

2. STOCHASTIC SUBSPACE IDENTIFICATION ALGORITHM

Data driven Stochastic Subspace Identification (SSI-Data), used in this paper, is a well-established OMA algorithm. It is described briefly in this section to underline differences in its mathematical formulation with respect to INOPMA technique. For more details on SSI-Data, interested readers are referred to [1-3]. Eq. (1) represents the equation of motion for a linear, time invariant, multiple degrees-of-freedom (MDOF) system.

$$\mathbf{M}\,\ddot{\mathbf{x}}(t)+\mathbf{C}\,\dot{\mathbf{x}}(t)+\mathbf{K}\,\mathbf{x}(t)=\mathbf{f}(t) \tag{1}$$

where **M** is mass matrix,**C** is damping matrix,**K** is stiffness matrix,$\boldsymbol{x}(t)$ is response vector and$\boldsymbol{f}(t)$ is force vector.The system described above can be represented in the equivalent state space form, in discrete time domain, in following manner

$$\begin{aligned}\mathbf{y}_{\mathbf{k+1}} &= \mathbf{A}\mathbf{y}_{\mathbf{k}}\\ \mathbf{y}_{\mathbf{k}} &= \mathbf{C}\mathbf{x}_{\mathbf{k}}\end{aligned} \tag{2}$$

where $\mathbf{y}_{\mathbf{k}}$ is vector of state variables at time instant k, matrix **A** is state transition matrix, containing all the necessary information about dynamic characteristics of the system and **C** is output matrix, relating output responses to state variables. It should be noted that in Eq. 2), process and measurement noise are not considered. The essence of SSI-Data algorithm lies in identification of state transition matrix **A** based on measured responses $\boldsymbol{x}$.

The starting point of SSI-Data is formation of Block Hankel data matrix $\mathbf{X}_h$, partitioned in two halves (as shown below), upper half referred as 'past' and lower as 'future'.

$$\mathbf{X}_h=\begin{bmatrix}\mathbf{x}_{(1:N-2m)}\\ \mathbf{x}_{(2:N-2m+1)}\\ \vdots\\ \mathbf{x}_{(2m:N)}\end{bmatrix}=\begin{bmatrix}\mathbf{X}_{hp}\\ \mathbf{X}_{hf}\end{bmatrix} \tag{3}$$

In Eq. (3), m represents the order of the system. Size of this matrix is ($2mN_o$ X N-$2m$), where N_o is the total number of output responses measured. The future responses, $\mathbf{X}_{hf}$, are now projected onto the past responses, $\mathbf{X}_{hp}$, as following

$$\mathbf{P}_m=\mathbf{X}_{hf}\mathbf{X}_{hp}^T\left(\mathbf{X}_{hp}\mathbf{X}_{hp}^T\right)^{-1}\mathbf{X}_{hp} \tag{4}$$

It should be noted that in practice, instead of using all responses, the future responses, , $\mathbf{X}_{hf}$, are projected onto only a subset of past responses, $\mathbf{X}_{hpr}$. These chosen responses ($\mathbf{X}_{hpr}$) are called *Reference* or *Projection* responses and in this form SSI is referred as *Reference based SSI*. In either case (whether all responses are used or a subset is used), the steps to follow remains same. More details of this algorithm are provided in [4].

$\mathbf{P}_m$ is related to the Observability matrix $\mathbf{O}_m$ and state vector sequences $\mathbf{Y}_m$

$$\mathbf{P}_m = \mathbf{O}_m \mathbf{Y}_m \tag{5}$$

Since the observability matrix, $\mathbf{O}_m$, is related to state transition matrix $\mathbf{A}$ and output matrix $\mathbf{C}$ (see Eq. 6), its estimation paves the way for identification of these matrices and in turn the modal parameters.

$$\mathbf{O}_m = \begin{pmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^m \end{pmatrix} \tag{6}$$

Thus, once $\mathbf{P}_m$ is estimated, as in Eq. (4), next task is to estimate Observability matrix $\mathbf{O}_m$, which is done using the singular value decomposition of $\mathbf{P}_m$.

$$\begin{aligned} \mathbf{P}_m &= \mathbf{U}\,\mathbf{S}\,\mathbf{V}^T \\ \mathbf{O}_m &= \mathbf{U}\,\mathbf{S}^{1/2} \end{aligned} \tag{7}$$

From Eq. (6), it follows that State transition matrix $\mathbf{A}$ and output matrix $\mathbf{C}$ can be estimated as

$$\begin{aligned} \mathbf{A} &= \mathbf{O}_{1:(m-1)} \mathbf{O}_{2:m}^{+} \\ \mathbf{C} &= \text{First block row of } \mathbf{O}_m \end{aligned} \tag{8}$$

where $\mathbf{O}_{1:(m-1)}$ is obtained by deleting the last block row of $\mathbf{O}_m$ and $\mathbf{O}_{2:m}$ is observability matrix upper-shifted by one block (in other words, matrix obtained after removing the first block of rows). $^{+}$ represents pseudo-inverse.

To obtain modal parameters, eigenvalue decomposition of state transition matrix $\mathbf{A}$ is performed, to explore the following relationship.

$$\mathbf{A} = \mathbf{\Psi}\,\mathbf{\Sigma}\,\mathbf{\Psi}^{-1} \tag{9}$$

where diagonal matrix $\mathbf{\Sigma}$ contains discrete-time system poles and mode shape ($\mathbf{\Phi}$) information can be extracted from $\mathbf{\Psi}$ using output matrix $\mathbf{C}$ as $\mathbf{\Phi} = \mathbf{C}\,\mathbf{\Psi}$.

3. INOPMA: THE CASE OF A SDOF SYSTEM

Consider a SDOF system with undamped natural frequency ω_n and damping ratio ζ, excited with a random force with spectrum of amplitude S_0. The correlation sequence of the output $y(t)$ is given by [1]

$$R(\tau) = \frac{S_0 \omega_n}{4\zeta} e^{-\zeta\omega_n\tau} \left[\cos(\omega_d \tau) + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin(\omega_d \tau) \right] \tag{10}$$

where $\omega_d = \omega_n \sqrt{1-\zeta^2}$.

This correlation sequence is a decaying sinusoid but with a certain phase shift that depends on the damping ratio. Therefore, it may be considered as an impulse response except that the phase shift needs to be taken into account.
Let us consider now the convolution of this correlation sequence with a pure sine wave with driving frequency ω, that is $f(t) = \sin(\omega t)$ and call the result $x(t)$, which is also a sine wave at the same frequency.

The transfer function $G(s) = \dfrac{x(s)}{f(s)}$ is given by [1]

$$G(s) = \frac{S_0}{4\zeta} \frac{s + 2\varsigma\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{11}$$

The tangent of the phase angle of this transfer function can be shown to be

$$\tan(\phi) = -\frac{\omega}{2\zeta\omega_n^3}\left[\omega^2 - \omega_n^2(1-4\zeta^2)\right]12 \tag{12}$$

This phase angle is exactly zero at the following frequency

$$\omega^* = \omega_n\sqrt{1-4\zeta^2} \tag{13}$$

By varying the driving frequency ω and computing the phase angle between the input and the output, this frequency can be identified exactly. We call this property the phase resonance. This is similar to the phase resonance property used in modal appropriation. The convolution done between the force and the correlation sequence may be thought of as applying a harmonic force to a certain structure.

Notice however since the damping ratio is not known, it is not possible at this stage to identify ω_n. It is possible however to identify ω^* exactly as this is the frequency at which the phase becomes zero. Next, the damping ratio needs to be estimated. With ζ known Eq. (13) yields an estimate of ω_n.

Once ω^* is identified, let us take the convolution of the correlation sequence with a harmonic signal that is in quadrature of phase (at pi/2) with respect to the first one and with amplitude α, that is $g(t) = (1 + j\alpha) f(t)$.

We can show that the phase resonance now occurs at the frequency $\tilde{\omega}^*$ given by

$$\tilde{\omega}^* = \omega^*\sqrt{1 + \alpha\zeta\beta(\zeta)} \tag{14}$$

where $\beta(\zeta)$ is given by

$$\beta(\zeta) = (1-4\zeta^2)^{-\frac{3}{2}} \tag{15}$$

By varying α and computing the new phase resonance frequencies, the damping ratio can be estimated by solving the following equation

$$\frac{1}{\omega^{*2}}\frac{\partial\tilde{\omega}^{*2}}{\partial\alpha} = \xi\beta(\zeta) \tag{16}$$

For exact estimation of ζ one can fit Eq. (16). However, for a wide range of damping ratios, the function $\beta(\zeta)$ is almost 1. A very good estimate of ζ may be given by

$$\zeta \approx \frac{1}{\omega^{*2}} \frac{\partial \tilde{\omega}^{*2}}{\partial \alpha} \tag{17}$$

Once the damping ratio is estimated, the natural frequency may be estimated from Eq. (13).

4. RESULTS

This section discusses the results of application of INOPMA algorithm to an analytical system and compares the estimated modal parameters with the theoretical modal parameters and those obtained using SSI. The analytical system used for this study is a 15 DOF system as shown in figure 1. Masses 1-10 have values 10/386.09 Kg and masses 11-15 have 0.5/386.09 Kg. All springs are 1000 N/m. Dampers C_1 and C_2 are 0.20 kg/s and C_3 is 0.05 kg/s. The system is excited by white random uncorrelated forces at all DOFs and the simulated time histories are sampled at a rate of 1024 Hz.

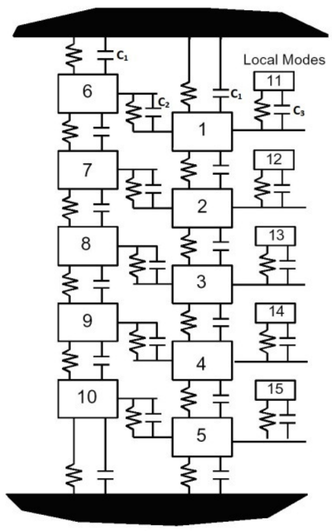

Figure 1. Analytical 15 Degrees-of-freedom System

Both SSI and INOPMA are applied to the response time histories. While applying SSI, DOFs 3, 11, 14 and 15 are used as projection responses. For INOPMA, all the time history is used to compute 512 correlation lags. This number of lags should be considered as a design parameter. It should not be too small as to miss some of the modes and it should not be too big to include noise. The current version of INOPMA uses a single output. In this study, output 5 is used. First, the driving frequency is incremented while computing the input/output angle. The frequency increment used is 0.01Hz. When the angle crosses zero, the characteristic frequency is found. As a second step, the damping ratio is found using the approach described in section 3. It is noticed that the estimation of the damping ratio is more delicate than the frequency and this may be considered as a drawback of the method, especially since the natural frequency estimate depends on the damping ratio estimate. One of the advantages of INOPMA is that it does not require a stabilizing diagram as it is the case for SSI algorithms and one does not have to worry about spurious modes, mode alignments, etc.

Table 1 shows a comparison of modal parameters estimated using the two algorithms along with the theoretical modes of the 15 DOF system. It is clear from the table that both algorithms perform equally well although a more thorough investigation similar to [1] is needed in order to make better conclusions.

Table 1. Comparison of results.

Mode	Frequency (Hz)			Damping (%)		
	Theoretical	SSI	INOPMA	Theoretical	SSI	INOPMA
1	15.99	15.99	16.01	1.004	1.196	1.32
2	30.86	30.83	31.21	1.937	2.085	1.87
3	43.6	43.61	42.62	2.735	2.68	2.74
4	46.44	46.46	46.62	2.912	2.969	3.10
5	53.32	53.4	53.54	3.337	3.414	3.24
6	53.39	53.49	53.59	3.345	3.198	3.51
7	59.41	59.58	59.32	3.714	3.779	3.78
8	61.62	61.57	61.71	3.858	4.115	4.18
9	68.81	68.82	68.76	4.298	5.172	4.78
10	73.63	73.6	73.58	4.592	4.493	4.32
11	128.8	128.7	129.05	2.609	2.691	2.55
12	136.6	136.6	137.08	2.455	2.473	2.51
13	143.9	143.9	143.8	2.329	2.372	2.45
14	150.8	150.9	151.18	2.221	2.245	2.32
15	157.5	157.5	157.18	2.122	2.142	2.31

5. CONCLUSIONS

We investigated in this paper the performance of the INOPMA algorithm [1] for in-operation modal identification. The method is compared on an analytical 15 dof simulation example and it is shown that both methods perform equally well. One of the advantages of INOPMA is that it isolates the modes mode by mode and therefore does not require a stabilization diagram similar to SSI algorithms. Moreover, there is no worries about spurious modes, alignments, etc. On the other hand, one of the disadvantages of INOPMA is in the dependency of the natural frequency on the damping ratio, which is not very robustly identified.

The extension of INOPMA to the MIMO case is on going.

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